Local entropy averages and the fine structure of measures

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University of Helsinki, Finland

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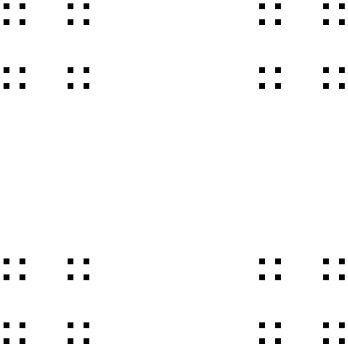
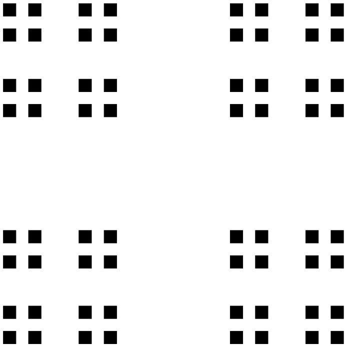
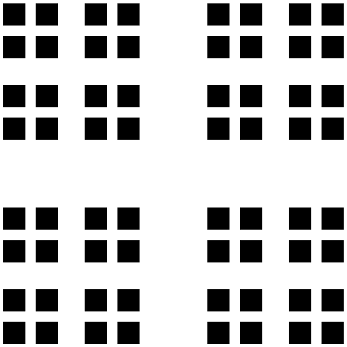
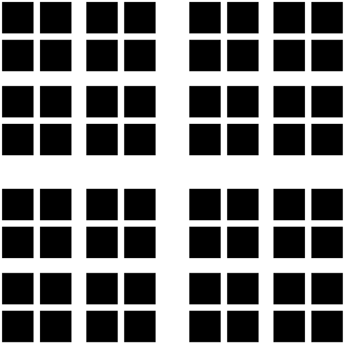
26.4.2012

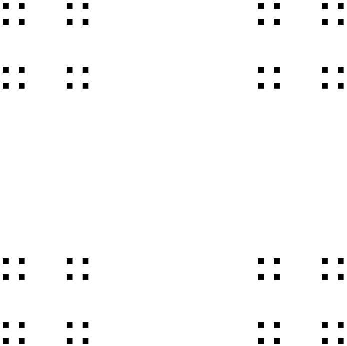
JOINT WORK WITH PABLO SHMERKIN AND VILLE SUOMALA

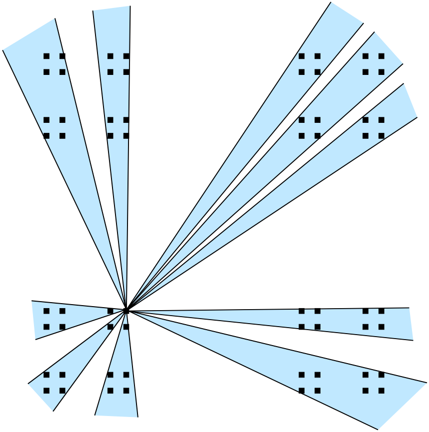


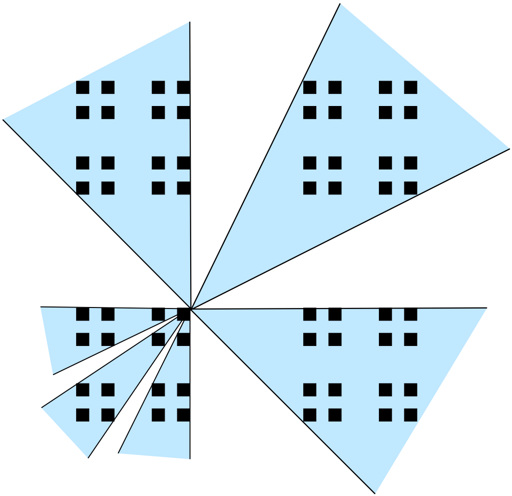


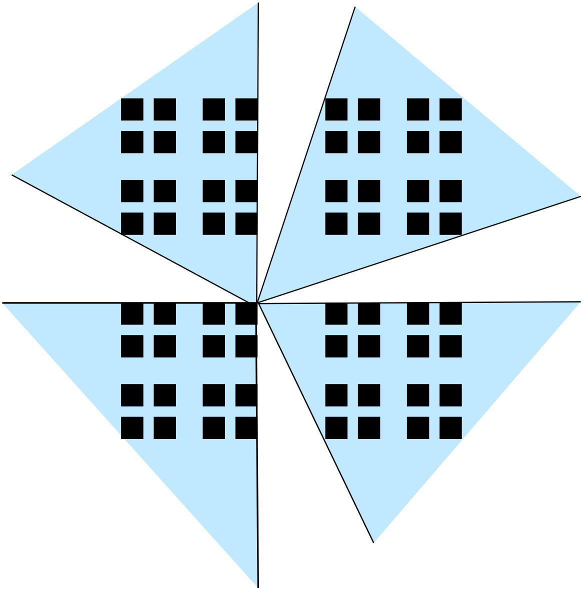


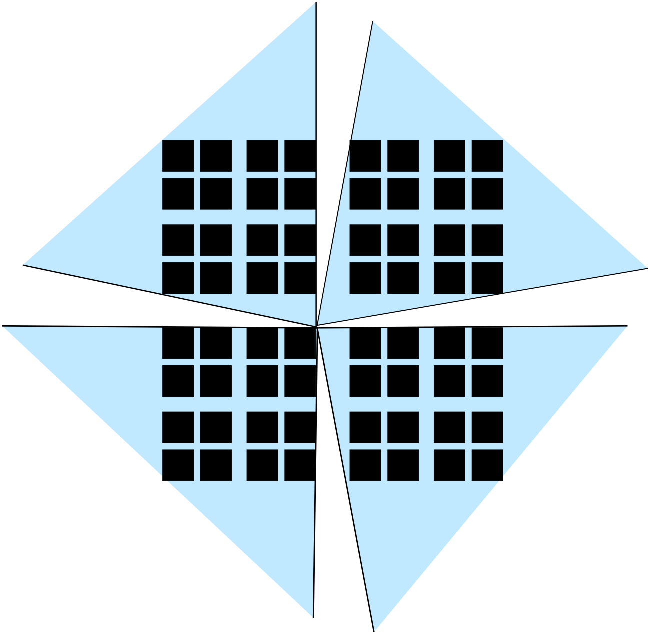












Heuristics

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How to make this precise?

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Geometry

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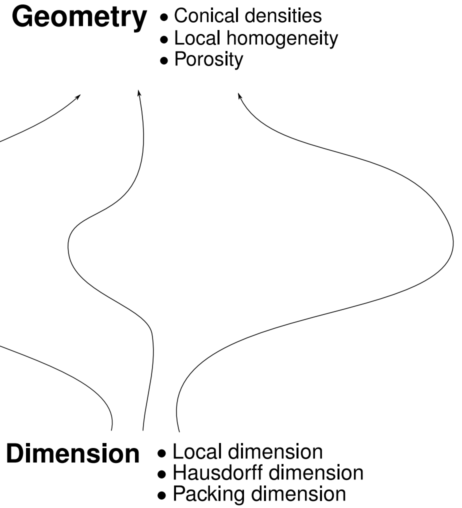
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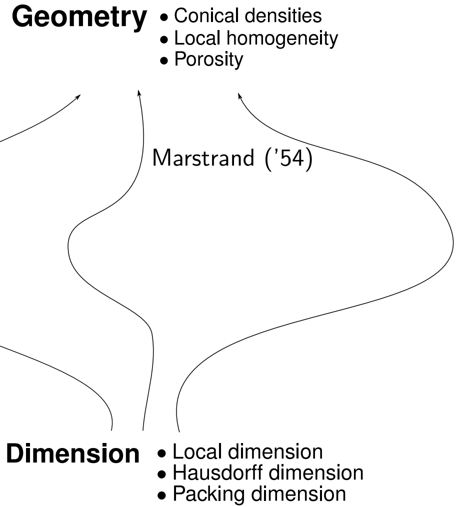
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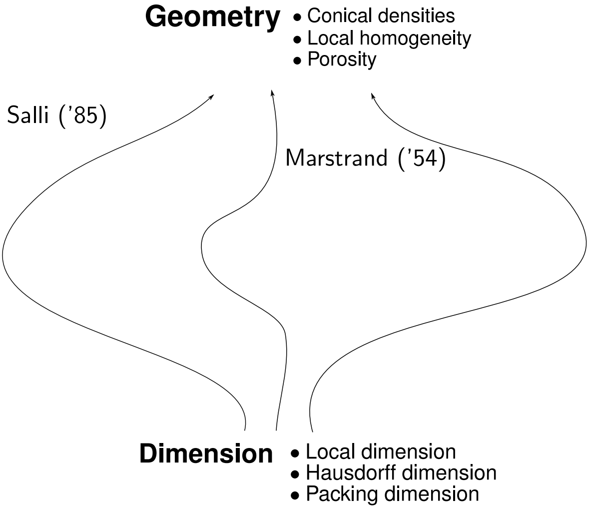

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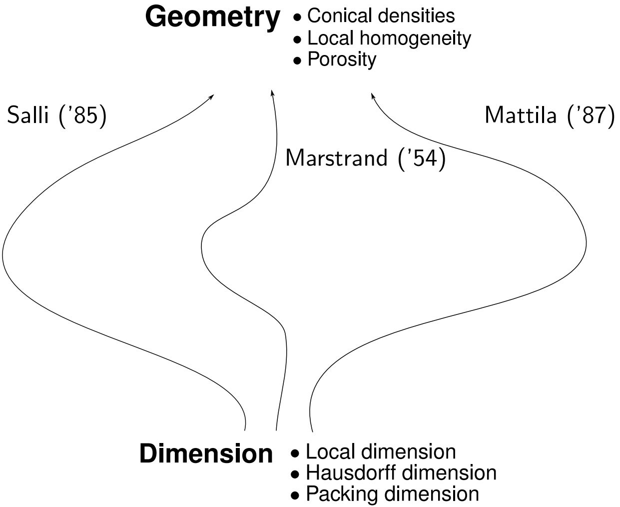
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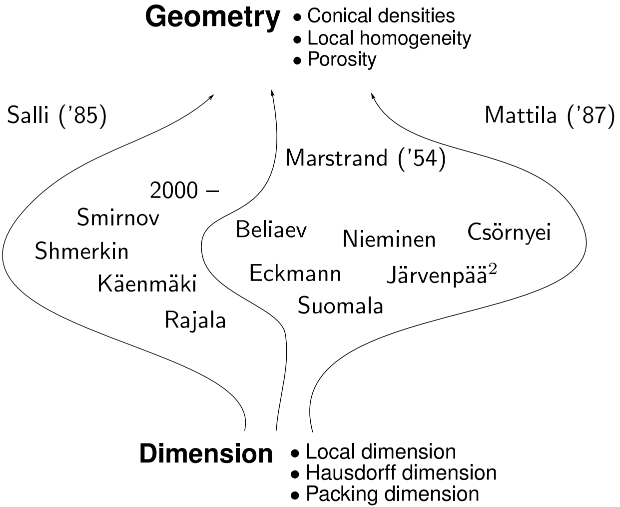
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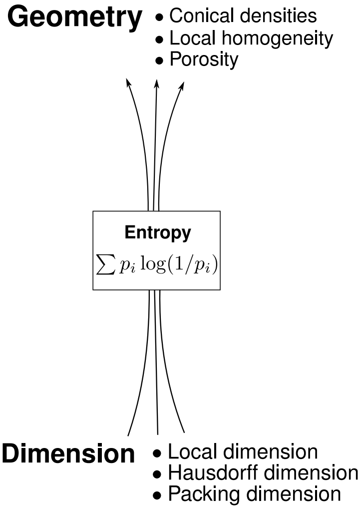
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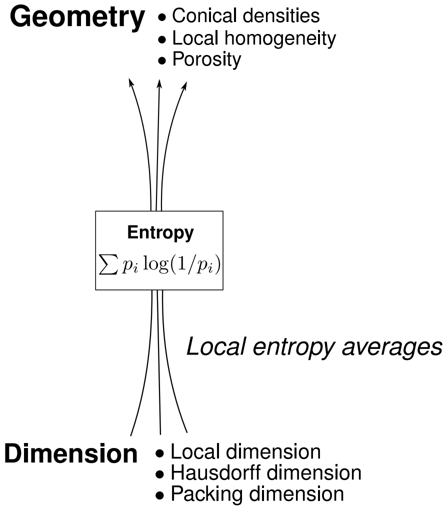

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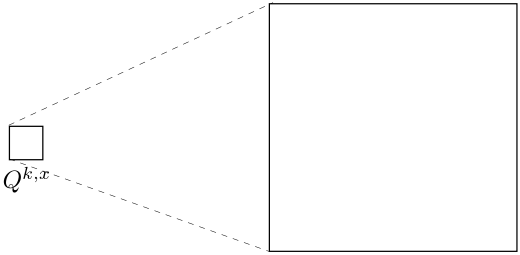

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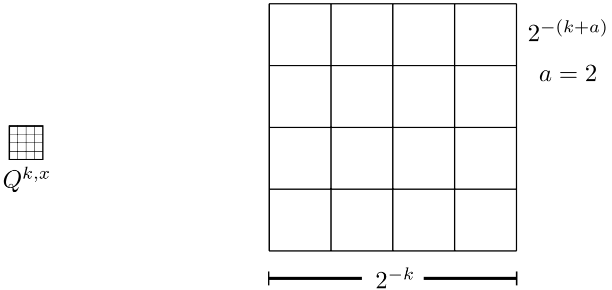

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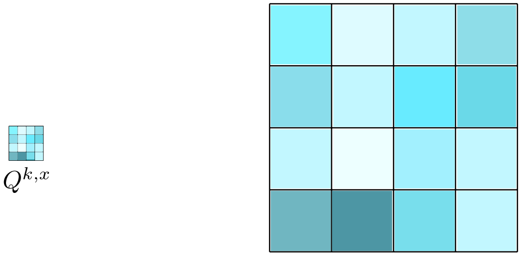

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$$\begin{aligned} \mathrm{por}_m(\mu, x, r, \varepsilon) = \sup\{\varrho > 0 : \; & \exists y_1, \ldots, y_m \in \mathbb{R}^d \text{ with} \\ & (y_i - x) \cdot (y_j - x) = 0, i \neq j, \\ & B(y_i, \varrho r) \subset B(x, r), \text{ and} \\ & \mu(B(y_i, \varrho r)) \leq \varepsilon \mu(B(x, r))\}. \end{aligned}$$

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- μ is **lower mean** (m, α, p)**-porous** at x, if for any $\varepsilon > 0$ and for all large enough $N \in \mathbb{N}$ we have

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References

- M. HOCHMAN, P. SHMERKIN: **Local entropy averages and projections of fractal measures** (2009), Annals of Mathematics (to appear), preprint at arXiv:0910.1956
- P. SHMERKIN: **The dimension of weakly mean porous measures: a probabilistic approach** (2010), International Mathematical Research Notices (to appear), preprint at arXiv:1010.1394
- T. SAHLSTEN, P. SHMERKIN, V. SUOMALA: **Dimension, entropy, and the local distribution of measures** (2011), submitted, preprint at arXiv:1110.6011